Set Operations and Duality Principle

Let A and B be subsets of the universal set U.

Intersection of A and B

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

Union of A and B

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$

(Relative) complement of A in B

$$B - A = \{x \mid x \in B \text{ and } x \notin A\}.$$

Complement of A

$$A^c = \{ x \mid x \in U \text{ and } x \notin A \}.$$

Symmetric difference of A and B

$$A \,\Delta\, B = (A - B) \cup (B - A).$$

Cartesian product of A and B

$$A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$$

Let E be an equation of set algebra. The dual E^* of E is the equation obtained by replacing each occurrence of $\cup$, $\cap$, U, $\varnothing$, in E by $\cap$, $\cup$, $\varnothing$, U, respectively.

Duality Principle: If any equation E is an identity, then its dual E^* is also an identity.

Laws of the Algebra of Sets – Duality Principle

Let A , B and C be subsets of the universal set $\boldsymbol{U}$.

$A \subseteq A \cup B$; $B \subseteq A \cup B$; $A \cap B \subseteq A$; $A \cap B \subseteq B$

$A - B = A \cap B^c$

$A - A = \varnothing$; $\varnothing - A = \varnothing$; $A - \varnothing = A$; $A - \boldsymbol{U} = \varnothing$

$A \Delta B = (A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

$A \Delta A = \varnothing$; $A \Delta \varnothing = A$; $A \Delta \boldsymbol{U} = \boldsymbol{U} - A = A^c$

Involution Law

$(A^c)^c = A$

Commutative Laws

$A \cup B = B \cup A$	$A \cap B = B \cap A$
$A \Delta B = B \Delta A$	

Associative Laws

$A \cup (B \cup C) = (A \cup B) \cup C$	$A \cap (B \cap C) = (A \cap B) \cap C$
$A \Delta (B \Delta C) = (A \Delta B) \Delta C$	

Identity Laws

$A \cup \varnothing = A$; $A \cup \boldsymbol{U} = \boldsymbol{U}$	$A \cap \boldsymbol{U} = A$; $A \cap \varnothing = \varnothing$

Idempotent Laws

$A \cup A = A$	$A \cap A = A$

Complement Laws

$A \cup A^c = \boldsymbol{U}$; $\boldsymbol{U}^c = \varnothing$	$A \cap A^c = \varnothing$; $\varnothing^c = \boldsymbol{U}$

Absorption Laws

$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

Distributive Laws

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

DeMorgan’s Laws

$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$
$A - (B \cup C) = (A - B) \cap (A - C)$	$A - (B \cap C) = (A - B) \cup (A - C)$